Journal of
Computational and Theoretical Nanoscience
Vol. 12, 1–7, 2015

Two-Temperature Generalized Thermoelastic Interaction of Functional Graded Material

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The aim of the present work is concerned with the solution of a problem on two-temperature generalized thermoelasticity for a functional graded material. The governing equations of two-temperature generalized thermoelasticity with one relaxation time for functionally graded materials (FGM) (i.e., material with spatially varying material properties) are established. Those equations are expressed in Laplace transform domain. The analytical solution in the transform domain is obtained by using the eigenvalue approach. Numerical results for the temperature distribution, displacement and thermal stress represented graphically.

Keywords: Two-Temperature, Functionally Graded Materials, Generalized Thermoelasticity, Eigenvalue Approach.

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1. INTRODUCTION

Functionally graded materials (FGMs) are composite materials formed of two or more constituent phases with a continuously variable composition. FGMs possess a number of advantages that make them attractive in potential applications, including a potential reduction of in-plane and transverse through-the-thickness stresses, an improved residual stress distribution, enhanced thermal properties, higher fracture toughness, and reduced stress intensity factors.[1]

Functionally graded materials (FGMs) can be viewed as an inhomogeneous materials with spatial form whose properties are function on spatial coordinates. Due to the continuous change of material properties in space, the absence of interfaces between different constituents or phases largely reduces the degree of material property mismatch and brings appealing physical behaviors superior to homogeneous and conventional materials. FGMs can be applied to many engineering structures subjected to severe thermal loadings such as high temperature and thermal shocks to reduce thermal stresses and suffer less thermal damage.[2]

Two models have been used to characterize the material gradation. One is the so-called continuum model, in which analytical functions such as exponent and power-law functions are commonly used to describe the continuously varying material properties. Although the continuum model may not be physical in practice, this model is convenient for conducting mathematical analysis. The other is the micromechanics model, which takes into account interactions between constituent phases and uses a certain representative volume element (RVE) to estimate the average local stress and strain fields of the composite, after which the local average fields are used to evaluate the effective material properties. The Mori-Tanaka method[3] and the self-consistent method[4] are two representatives of these models. In this paper, attention is focused on the continuum model only.

Mathematically, the thermoelastic analysis in FGMs is described by partial differential equations with variable coefficients, to which a closed-form analytical solution is difficult to obtain and is available for limited problems with simple geometries, certain types of gradation of material properties, specific types of boundary conditions and special loading cases. So, numerical methods have been developed for investigating static or dynamic problems mainly involving the evaluation of temperature field and stress fields to reduce dependency on costly and time consuming experimental analysis. Among the established numerical methods, the finite element method (FEM)[5–7] or the graded finite element method,[8,9] the boundary element method (BEM) or boundary integral equation method (BIEM)[10–12] are most versatile to deal with thermoelastic

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analysis. More recently, as alternatives to the FEM and BEM, meshless methods have been used for thermal analysis of FGMs. The corresponding developments in thermal and stress computation in FGMs include: Rao and Rahman[13] used element-free Galerkin method (EFGM) to simulate stress fields near the crack tip in FGMs. The same method was used by Dai et al.[14] to study thermomechanical behavior of FGM plates. Ching and Yen[15,16] analyzed the static and transient responses of FGMs under mechanical and thermal loads by means of the meshless local Petrov–Galerkin (MLPG) method.[17,18] Moreover, Sladek et al. solved dynamic anti-plane shear crack problem and transient heat conduction in FGMs by a meshless local boundary integral equation (LBIE) method.[19,20]

Chen and Gurtin,[21] Chen et al.[22,23] have formulated a theory of heat conduction in deformable bodies, which depends upon two distinct temperatures, the conductive temperature φ and the thermo-dynamical temperature T. For time independent situations, the difference between these two temperatures is proportional to the heat supply, and in the absence of any heat supply, the two temperatures are identical Chen and Gurtin.[21] For time dependent problems, however, and for wave propagation problems in particular, the two temperatures are in general different regardless of the presence of a heat supply. The two temperatures T, φ and the strain are found to have representations in the form of a traveling wave plus a response, which occurs instantaneously throughout the body,[24] and Warren and Chen[25] investigated the wave propagation in the two-temperature theory of thermoelasticity.

Youssef[26] investigated two-temperature generalized thermoelasticity theory together with a general uniqueness theorem and solved many applications in the context of this theory in Refs. [27–30]. During the last three decades, a number of investigations[31–59] have been carried out using the aforesaid theories of generalized thermoelasticity. Recently,[60–62] variants problems in waves are studied. Other forms are described for example in the Refs. [63–65].

This work is concerned with the solution of a problem on two-temperature generalized thermoelasticity for a functional graded material. The governing equations of two-temperature generalized thermoelasticity with one relaxation time for functionally graded materials (FGM) will be established. Those equations will be expressed in Laplace transform domain. The analytical solution in the transform domain will be obtained by using the eigenvalue approach. Numerical results for the temperature distribution, displacement and thermal stress will be represented graphically.

2. BASIC EQUATION

Equation of motion

$$\sigma_{ij,j} = \rho \frac{\partial^2 u_i}{\partial t^2} \tag{1}$$

Equation of heat conduction

$$(K\varphi_{,i})_{,i} = \left(\frac{\partial}{\partial t} + \tau \frac{\partial^2}{\partial t^2}\right)(\rho c_e T + \gamma T_0 e) \tag{2}$$

and

$$\varphi - T = a\varphi_{ii}, \quad i = x, y, z \tag{3}$$

The constitutive equations are given by

$$\sigma_{ij} = 2\mu e_{ij} + [\lambda e - \gamma (T - T_0)]\,\delta_{ij} \tag{4}$$

and

$$e = e_{ii}, \quad i = x, y, z \tag{5}$$

where λ, μ are the Lame's constants; ρ is the density of the medium; c_e is the specific heat at constant strain; α_t is the coefficient of linear thermal expansion; t is the time; φ is the conductive temperature; a is non-negative constant which is called two-temperature parameter; T is the thermodynamic temperature change of a material particle; T_0 is the reference temperature; τ is the relaxation time; K is the thermal conductivity; δ_{ij} is the Kronecker symbol; σ_{ij} are the components of stress tensor; u_i are the components of displacement vector.

Thus, we replace λ, μ, γ, K and ρ by $\lambda_0 f(\mathrm{X})$, $\mu_0 f(\mathrm{X})$, $\gamma_0 f(\mathrm{X})$, $K_0 f(\mathrm{X})$ and $\rho_0 f(\mathrm{X})$ where λ_0, μ_0, γ_0, K_0 and ρ_0 are assumed to be constants and $f(\mathrm{X})$ is a given dimensionless function of the space variable $\mathrm{X} = (x, y, z)$. Then the Eqs. (1) to (3) take the following form:

$$\begin{aligned} f(\mathrm{X})\left[2\mu_0 e_{ij} + [\lambda_0 e - \gamma_0 (T - T_0)]\,\delta_{ij}\right]_{,j} \\ + f(\mathrm{X})_{,j}\left[2\mu_0 e_{ij} + [\lambda_0 e - \gamma_0 (T - T_0)]\,\delta_{ij}\right] \\ = \rho_0 f(\mathrm{X}) \frac{\partial^2 u_i}{\partial t^2} \end{aligned} \tag{6}$$

$$\left(K_o f(\mathrm{X})\varphi_{,i}\right)_{,i} = \left(\frac{\partial}{\partial t} + \tau \frac{\partial^2}{\partial t^2}\right)(\rho_o f(\mathrm{X}) c_e T + \gamma_o f(\mathrm{X}) T_0 e) \tag{7}$$

$$\sigma_{ij} = f(\mathrm{X})\left[2\mu_o e_{ij} + [\lambda_o e - \gamma_o (T - T_0)]\,\delta_{ij}\right] \tag{8}$$

3. FORMULATION OF THE PROBLEM

Let us consider a functionally graded isotropic thermoelastic body at a uniform reference temperature T_0, occupying the region $x \geq 0$ where the x-axis is taken perpendicular to the bounding plane of the half-space pointing inwards. It assumed that the state of the medium depends only on x and the time variable t, so that the displacement vector $\vec{u}$ and temperatures field T and φ can be expressed in the following form:

$$\vec{u} = (u(x,t), 0, 0), \quad T = T(x,t), \quad \varphi = \varphi(x,t) \tag{9}$$

It is assumed that the material properties depend only on the x-coordinate. So, we take $f(\mathrm{X})$ as $f(x)$. In the context of the generalized thermoelasticity theory based on

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the Lord and Shulman model, the equation of motion, heat equation, and constitutive equation can be written as:

$$f(x)\left[(\lambda_0+2\mu_0)\frac{\partial^2 u}{\partial x^2}-\gamma_0\frac{\partial T}{\partial x}\right] + \frac{\partial f(x)}{\partial x}\left[(\lambda_0+2\mu_0)\frac{\partial u}{\partial x}-\gamma_0 T\right]=\rho_0 f(x)\frac{\partial^2 u}{\partial t^2} \quad (10)$$

$$K_0\left(f(x)\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial f(x)}{\partial x}\frac{\partial\varphi}{\partial x}\right) = f(x)\left(\frac{\partial}{\partial t}+\tau\frac{\partial^2}{\partial t^2}\right)\left(\rho_0 c_e T+\gamma_0 T_0\frac{\partial u}{\partial x}\right) \quad (11)$$

$$\varphi-T=a\frac{\partial^2\varphi}{\partial x^2} \quad (12)$$

and

$$\sigma_{xx}=f(x)\left[(\lambda_o+2\mu_o)\frac{\partial u}{\partial x}-\gamma_o(T-T_0)\right] \quad (13)$$

We define the following dimensionless quantities

$$(x',u')=\frac{c}{\chi}(x,u),\quad T'=\frac{T-T_0}{T_0},\quad \varphi'=\frac{\varphi-T_0}{T_0}$$
$$(t',\tau'')=\frac{c^2}{\chi}(t,\tau),\quad \sigma'_{xx}=\frac{\sigma_{xx}}{\lambda_0+2\mu_0},\quad a'=\frac{\chi^2}{c^2}a$$

where $c^2=\lambda_0+2\mu_0/\rho_0$ and $\chi=K_o/\rho_o c_e$.

Upon introducing in Eqs. (10)–(13), and after suppressing the primes, we obtain

$$f(x)\left[\frac{\partial^2 u}{\partial x^2}-\eta\frac{\partial T}{\partial x}\right]+\frac{\partial f(x)}{\partial x}\left[\frac{\partial u}{\partial x}-\eta T\right]=f(x)\frac{\partial^2 u}{\partial t^2} \quad (14)$$

$$f(x)\frac{\partial^2 T}{\partial x^2}+\frac{\partial f(x)}{\partial x}\frac{\partial T}{\partial x}=f(x)\left(\frac{\partial}{\partial t}+\tau\frac{\partial^2}{\partial t^2}\right)\left(T+\varepsilon\frac{\partial u}{\partial x}\right) \quad (15)$$

$$\varphi-T=a\frac{\partial^2\varphi}{\partial x^2} \quad (16)$$

$$\sigma_{xx}=f(x)\left[\frac{\partial u}{\partial x}-\eta T\right] \quad (17)$$

where $\eta=T_0\gamma_o/\lambda_o+2\mu_o$, $\varepsilon=\gamma_o/\rho_o c_e$.

4. EXPONENTIAL VARIATION OF NON-HOMOGENEITY

We will consider

$$f(x)=e^{nx} \quad (18)$$

where n is dimensionless constant.[31]

Then, Eqs. (14)–(17) will reduce to

$$\left[\frac{\partial^2 u}{\partial x^2}-\eta\left(\frac{\partial\varphi}{\partial x}-a\frac{\partial^3\varphi}{\partial x^3}\right)\right] + n\left[\frac{\partial u}{\partial x}-\eta\left(\varphi-a\frac{\partial^2\varphi}{\partial x^2}\right)\right]=\frac{\partial^2 u}{\partial t^2} \quad (19)$$

$$\left(\frac{\partial^2\varphi}{\partial x^2}+n\frac{\partial\varphi}{\partial x}\right)=\left(\frac{\partial}{\partial t}+\tau\frac{\partial^2}{\partial t^2}\right)\left(\varphi-a\frac{\partial^2\varphi}{\partial x^2}+\varepsilon\frac{\partial u}{\partial x}\right) \quad (20)$$

$$\sigma_{xx}=e^{nx}\left[\frac{\partial u}{\partial x}-\eta\left(\varphi-a\frac{\partial^2\varphi}{\partial x^2}\right)\right] \quad (21)$$

5. APPLICATION

We assume that the medium is initially at rest. The undisturbed state is maintained at reference temperature.

Then, we have

$$u(x,0)=\frac{\partial u(x,0)}{\partial t}=0,\quad \varphi(x,0)=\frac{\partial\varphi(x,0)}{\partial t}=0 \quad (22)$$

We consider the problem of a thick plate of finite high l. Choosing the x-axis perpendicular to the surface of the plate with the origin coinciding with the lower plate, the region Ω under consideration becomes:

$$\Omega=\{(x,y,x):0\le x\le l,-\infty<y<\infty,-\infty<z<\infty\} \quad (23)$$

The surface of the plate is taken to be traction free. The lower plate is subjected to a thermal shock. The upper plate is kept at zero temperature. Mathematically these can be written

$$\sigma_{xx}(0,t)=0 \quad (24)$$

$$\varphi(0,t)=\varphi_1 H(t) \quad (25)$$

$$\sigma_{xx}(l,t)=0 \quad (26)$$

and

$$\varphi(l,t)=0 \quad (27)$$

where $H(t)$ denotes the Heaviside unit step function.

6. GOVERNING EQUATIONS IN THE LAPLACE TRANSFORM DOMAIN

Applying the Laplace transforms for Eqs. (19)–(21) and (24)–(27) define by the formula

$$\bar f(s)=L[f(t)]=\int_0^\infty f(t)e^{-st}dt \quad (28)$$

Hence, we obtain the following system of differential equations

$$\left[\frac{d^2\bar u}{dx^2}-\eta\left(\frac{d\bar\varphi}{dx}-a\frac{d^3\bar\varphi}{dx^3}\right)\right] + n\left[\frac{d\bar u}{dx}-\eta\left(\bar\varphi-a\frac{d^2\bar\varphi}{dx^2}\right)\right]=s^2 u \quad (29)$$

$$\left(\frac{d^2\bar\varphi}{dx^2}+n\frac{d\bar\varphi}{dx}\right)=\left(s+\tau s^2\right)\left(\varphi-a\frac{d^2\bar\varphi}{dx^2}+\varepsilon\frac{d\bar u}{dx}\right) \quad (30)$$

$$\bar\sigma_{xx}=e^{nx}\left[\frac{d\bar u}{dx}-\eta\left(\bar\varphi-a\frac{d^2\bar\varphi}{dx^2}\right)\right] \quad (31)$$

$$\bar\sigma_{xx}(0,s)=0 \quad (32)$$

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$$\bar{\varphi}(0,s)=\frac{\varphi_1}{s} \tag{33}$$

$$\bar{\sigma}_{xx}(l,s)=0 \tag{34}$$

and

$$\bar{\varphi}(l,s)=0 \tag{35}$$

Equations (29) and (30) can be written in a vector-matrix differential equation as follows:[66]

$$\frac{d\vec{V}}{dx}=A\vec{V} \tag{36}$$

where

$$\vec{V}=\left[\bar{u}\ \bar{\varphi}\ \frac{d\bar{u}}{dx}\ \frac{d\bar{\varphi}}{dx}\right]^T \tag{37}$$

$$A=\begin{bmatrix}0&0&1&0\\0&0&0&1\\a_{31}&a_{32}&a_{33}&a_{34}\\0&a_{42}&a_{43}&a_{44}\end{bmatrix} \tag{38}$$

and

$$a_{31}=\frac{s^2}{a_4},\ a_{32}=\frac{1}{a_4}\left(n\eta-\eta a a_{42}a_{44}-n\eta a a_{42}\right)$$
$$a_{33}=-\frac{1}{a_4}\left(n+\eta a a_{43}a_{44}+n\eta a a_{43}\right)$$
$$a_{34}=\frac{1}{a_4}\left(\eta-a\eta\left(a_{42}+a_{44}^2\right)-n\eta a a_{44}\right)$$
$$a_{42}=\frac{s(1+\tau s)}{1+as(1+\tau s)},\quad a_{43}=\frac{\varepsilon s(1+\tau s)}{1+as(1+\tau s)}$$
$$a_{44}=\frac{-n}{1+as(1+\tau s)},\quad a_4=1+aa_{43}\eta$$

7. SOLUTION OF THE VECTOR-MATRIX DIFFERENTIAL EQUATION

Let us now proceed to solve Eq. (36) by the eigenvalue approach proposed by Ref. [66]. The characteristic equation of the matrix A takes the form

$$\begin{aligned}&a_{31}a_{42}+(a_{33}a_{42}-a_{32}a_{43}+a_{31}a_{44})\lambda\\&+(a_{33}a_{44}-a_{31}-a_{42}-a_{34}a_{43})\lambda^2\\&-(a_{33}+a_{44})\lambda^3+\lambda^4=0\end{aligned} \tag{39}$$

The roots of the characteristic Eq. (39) which are also the eigenvalues of matrix A are of the form $\lambda=\lambda_1$, $\lambda=\lambda_2$, $\lambda=\lambda_3$, $\lambda=\lambda_4$.

The eigenvector $\vec{X}=[x_1,x_2,x_3,x_4]^T$, corresponding to eigenvalue λ can be calculated as:

$$\begin{aligned}&x_1=a_{32}+a_{34}\lambda,\quad x_2=-a_{31}+(\lambda-a_{33})\lambda\\&x_3=\lambda x_1,\quad x_4=\lambda x_2\end{aligned} \tag{40}$$

From Eq. (39), we can easily calculate the eigenvector $\vec{X}_j$, corresponding to eigenvalue λ_j, $j=1,2,3,4$.

For further reference, we shall use the following notations:

$$\begin{aligned}&\vec{X}_1=\left[\vec{X}\right]_{\lambda=\lambda_1},\quad \vec{X}_2=\left[\vec{X}\right]_{\lambda=\lambda_2}\\&\vec{X}_3=\left[\vec{X}\right]_{\lambda=\lambda_3},\quad \vec{X}_4=\left[\vec{X}\right]_{\lambda=\lambda_4}\end{aligned} \tag{41}$$

The solution of Eq. (36) can be written from as follows:

$$\begin{aligned}\vec{V}&=\sum_{j=1}^{4}B_j\vec{X}_je^{\lambda_i x}\\&=B_1\vec{X}_1e^{\lambda_1 x}+B_2\vec{X}_2e^{\lambda_2 x}+B_3\vec{X}_3e^{\lambda_3 x}+B_4\vec{X}_4e^{\lambda_4 x}\end{aligned} \tag{42}$$

where B_1, B_2, B_3, and B_4 are constants to be determined from the boundary condition of the problem.

Thus, the field variables can be written for x and s as:

$$\bar{u}(x,s)=\sum_{j=1}^{4}B_jx_3^je^{\lambda_j x} \tag{43}$$

$$\bar{\varphi}(x,s)=\sum_{j=1}^{4}B_jx_4^je^{\lambda_j x} \tag{44}$$

$$\bar{T}(x,s)=\sum_{j=1}^{4}B_jx_4^j\left(1-a\lambda_j^2\right)e^{\lambda_j x} \tag{45}$$

$$\bar{\sigma}_{xx}(x,s)=\sum_{j=1}^{4}(\lambda_jx_3^j-\eta x_4^j(1-a\lambda_j^2))B_je^{(\lambda_j+n)x} \tag{46}$$

To complete the solution we have to know the constants B_1, B_2, B_3, and B_4, by using the boundary conditions (32)–(35) we can obtain

$$\begin{pmatrix}B_1\\B_2\\B_3\\B_4\end{pmatrix}=\begin{pmatrix}H_{11}&H_{12}&H_{13}&H_{14}\\H_{21}&H_{22}&H_{23}&H_{24}\\H_{31}&H_{23}&H_{33}&H_{34}\\H_{41}&H_{42}&H_{43}&H_{44}\end{pmatrix}^{-1}\begin{pmatrix}0\\0\\\frac{\varphi_1}{s}\\0\end{pmatrix} \tag{47}$$

where

$$H_{11}=\lambda_1x_3^1-x_4^j\eta\left(1-a\lambda_1^2\right),\ H_{12}=\lambda_2x_3^2-x_4^2\eta\left(1-a\lambda_2^2\right)$$
$$H_{13}=\lambda_3x_3^3-x_4^3\eta\left(1-a\lambda_3^2\right),\ H_{14}=\lambda_4x_3^4-x_4^4\eta\left(1-a\lambda_4^2\right)$$
$$H_{21}=\left(\lambda_1x_3^1-x_4^1\eta\left(1-a\lambda_1^2\right)\right)e^{\lambda_1 l}$$
$$H_{22}=\left(\lambda_2x_3^2-x_4^2\eta\left(1-a\lambda_2^2\right)\right)e^{\lambda_2 l}$$
$$H_{23}=\left(\lambda_3x_3^3-x_4^3\eta\left(1-a\lambda_3^2\right)\right)e^{\lambda_3 l}$$
$$H_{24}=\left(\lambda_4x_3^4-x_4^4\eta\left(1-a\lambda_4^2\right)\right)e^{\lambda_4 l}$$
$$H_{31}=x_4^1,\ H_{32}=x_4^2,\ H_{33}=x_4^3,\ H_{34}=x_4^4$$
$$H_{41}=x_4^1e^{\lambda_1 l},\ H_{42}=x_4^2e^{\lambda_2 l}$$
$$H_{43}=x_4^3e^{\lambda_3 l},\ H_{44}=x_4^4e^{\lambda_4 l}$$

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8. NUMERICAL INVERSION OF THE LAPLACE TRANSFORMS

For the final solution of temperature, displacement and stress distributions in the time domain, we adopt a numerical inversion method based on the Stehfest.[67] In this method, the inverse $f(t)$ of the Laplace transform $f(s)$ is approximated by the relation

$$f(t) = \frac{\ln 2}{t} \sum_{j=1}^{N} V_j F\left(\frac{\ln 2}{t} j\right) \tag{48}$$

Where V_j is given by the following equation:

$$V_i = (-1)^{(N/2+1)} \sum_{k=i+1/2}^{\min(i,N/2)} \frac{k^{(N/2+1)}(2k)!}{\left(\frac{N}{2}-k\right)!k!(i-k)!(2k-1)!} \tag{49}$$

The parameter N is the number of terms used in the summation in Eq. (48) and should be optimized by trial and error. Increasing N increases the accuracy of the result up to a point, and then the accuracy declines because of increasing round-off errors. An optimal choice of $10 \le N \le 14$ has been reported by Lee et al. for some problem of their interest.[68] Thus, the solutions of all variables in physical space-time domain are given by:

$$u(x,t) = \frac{\ln 2}{t} \sum_{i=1}^{N} V_i \bar{u}\left(x, \frac{\ln 2}{t} i\right) \tag{50}$$

$$\varphi(x,t) = \frac{\ln 2}{t} \sum_{i=1}^{N} V_i \bar{\varphi}\left(x, \frac{\ln 2}{t} i\right) \tag{51}$$

$$T(x,t) = \frac{\ln 2}{t} \sum_{i=1}^{N} V_i \bar{T}\left(x, \frac{\ln 2}{t} i\right) \tag{52}$$

and

$$\sigma_{xx}(x,t) = \frac{\ln 2}{t} \sum_{i=1}^{N} V_i \bar{\sigma}_{xx}\left(x, \frac{\ln 2}{t} i\right) \tag{53}$$

9. NUMERICAL RESULTS AND DISCUSSION

The copper material was chosen for purposes of numerical evaluations and the constants of the problem were taken as following

$$\lambda_0 = 7.76 \times 10^{10} (kg)(m)^{-1}(s)^{-2}$$
$$\mu_0 = 3.86 \times 10^{10} (kg)(m)^{-1}(s)^{-2}$$
$$T_0 = 293(K), \quad K_0 = 3.68 \times 10^{2} (kg)(m)(K)^{-1}(s)^{-3}$$
$$c_e = 3.831 \times 10^{2} (m)^2(K)^{-1}(s)^{-2}, \quad \varphi_1 = 1, \quad l = 4$$
$$\rho_0 = 8.954 \times 10^{3} (kg)(m)^{-3}, \quad \alpha_t = 17.8 \times 10^{-6}(K)^{-1}$$
$$\tau = 0.05$$

Figures 1–5, present the conductive temperature distribution, the thermo-dynamical temperature distribution, the displacement distribution, the strain distribution and the

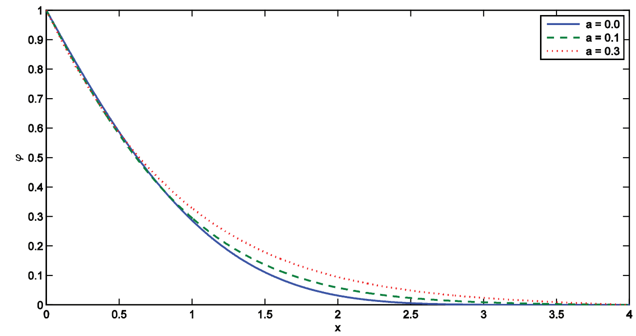

Fig. 1. The conductive temperature distribution with different values of two-temperature parameter.

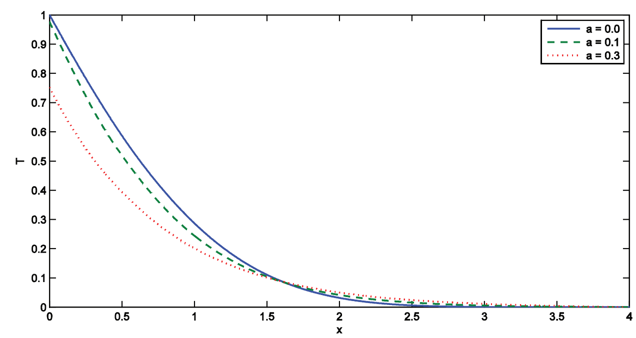

Fig. 2. The thermo-dynamical temperature distribution with different values of two-temperature parameter.

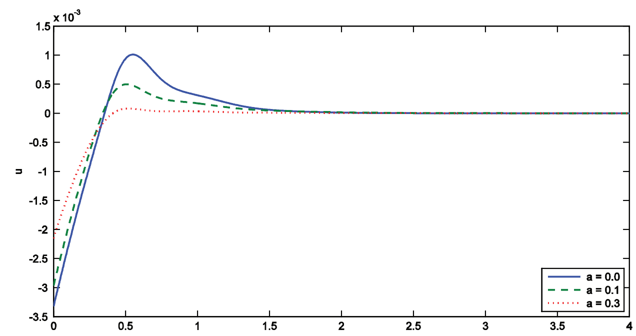

Fig. 3. The displacement distribution with different values of two-temperature parameter.

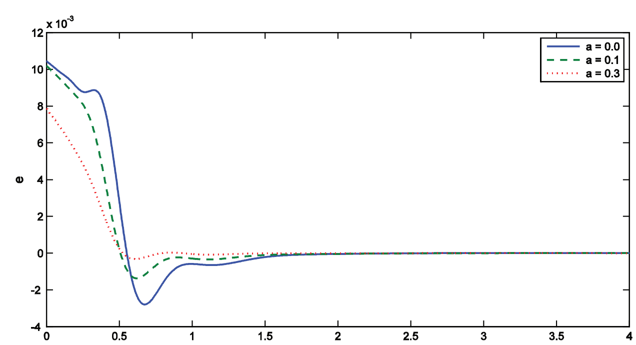

Fig. 4. The strain distribution with different values of two-temperature parameter.

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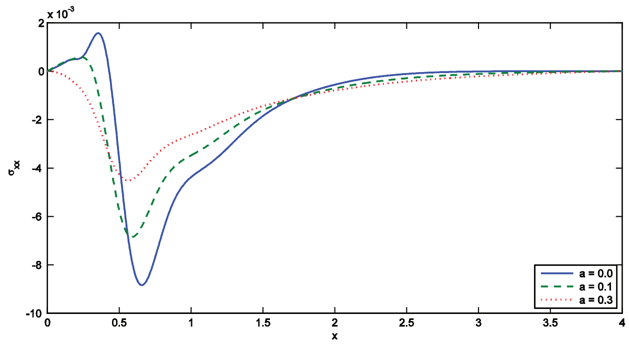

Fig. 5. The stress distribution with different values of two-temperature parameter.

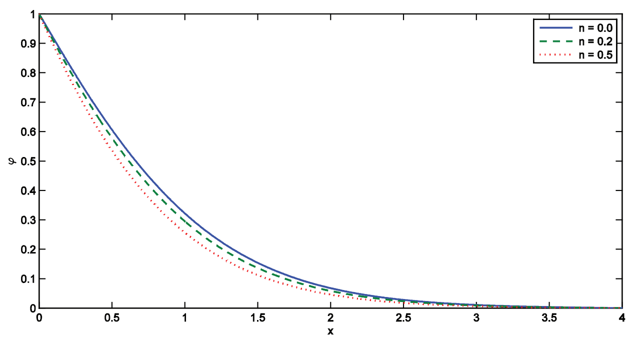

Fig. 6. The conductive temperature distribution with different values n.

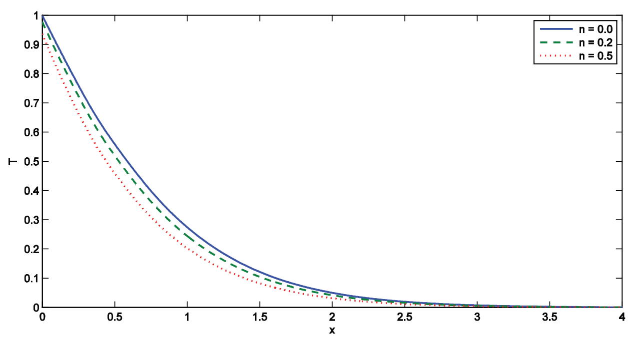

Fig. 7. The thermo-dynamical temperature distribution with different values n.

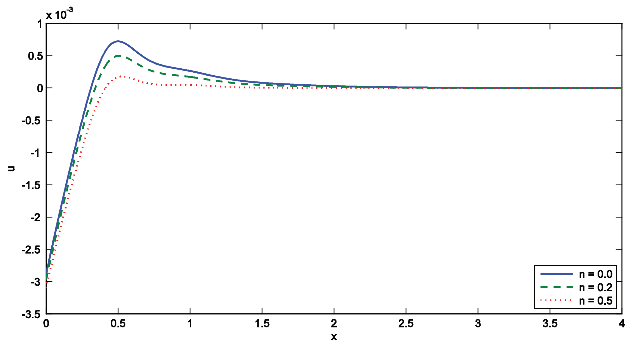

Fig. 8. The displacement distribution with different values n.

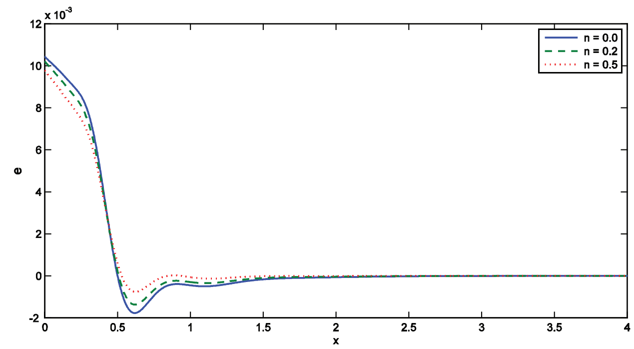

Fig. 9. The strain distribution with different values n.

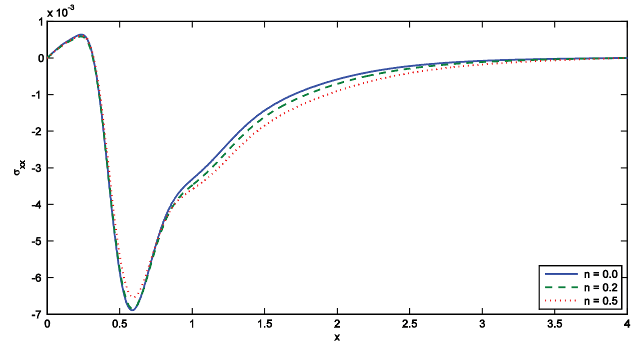

Fig. 10. The stress distribution with different values n.

stress distribution with different values of two-temperature parameter respectively. According to those figures, the two-temperature parameter has significant effects on all the studied fields.

Figures 6–10, present the conductive temperature distribution, the thermo-dynamical temperature distribution, the displacement distribution, the strain distribution and the stress distribution with different values of n parameter respectively. According to those figures, the n parameter has significant effects on all the studied fields.

10. CONCLUSIONS

The conductive temperature distribution, the thermo-dynamical temperature distribution, the displacement distribution, the strain distribution and the stress distribution all are depend on not only the dimension x and t but also on the two temperature parameter and the functional graded parameter.

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Received: 11 July 2014. Accepted: 28 July 2014.